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Mathematical Morphology and Medical Image Processing

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Abstract: *In the medical images such as X-ray, CT, there may be some kind of abnormal entities like cancer cells, tumor in such close proximity it is difficult to find out contrast and resolution between two entities hence diagnosis and treatment highly depends upon reliable and accurate finding of Radiologist. The medical image processing plays a crucial role in artificial detection and diagnosis of diseases along with automation of medical industry. The aim of medical image processing is to enhance contrast and resolution between closely packed entities on the basis of features like intensity value, color, texture so that visual perception can be increased. This paper introduces mathematical model of morphological operation of erosion and dilation operation with real time CT scan images with various structuring element. The erosion and dilatation operation on brain tumor images is performed with various structuring element. The results shows that affected area with tumor can be prominently distinguished from the original images obtained using CT scan.*

Keywords: *Mathematical Morphology, dilation and erosion*

I. Introduction

In the medical images such as X-ray, MRI, CT Scan, Endoscopy, there exist a large number of human organs with close proximity so it becomes difficult to diagnose clear and accurate shape, size and connectivity of any abnormal entity like tumor, micro calcification, tissue clustering etc[1,7]Mathematical morphology founds a tremendous and successful application for detection of shape, size and pattern of such abnormal entities in medical images which is helpful for diagnosis and treatment of patient. Mathematical morphology uses the concepts of set theory, lattice theory, topology and random functions for the feature extraction in an image while traditional image processing is based on the intensity of pixels [2,9,10] .Mathematical morphology based on the topology and geometrical continuous space which consist a set of operators like erosion, dilation, opening, closing skeletonization and conditional bisector that transforms the image according to size, shape, convexity, connectivity and geodesic distance[3,4]. Therefore mathematical morphology founds a great application for determining the shape, size and pattern reorganization of different close packed objects in images[3,10].

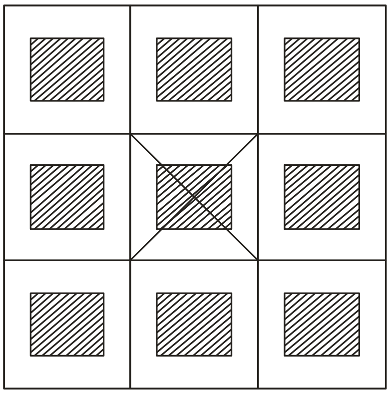
Mathematical Morphology determines the geometrical Structure of an image by probing it with small patterns called 'Structuring element' of varying size and shape. This predefined probe or Structure element is checked in each pixel that’s how does this shape matches or misses local shapes in the image for extracting features from images[5,6].

Mathematical morphology— An image can be defined as function of two real variables a (x, y) or two discrete variables a [m, n]. In other words an image consist of a set of either continuous or discrete coordinates. In a sense the set corresponds to the points of pixels that belongs to the image. Mathematical morphology basically deals with set theory for describing shapes of object in the images. It used to investigate the interaction between an image and a certain predefined structuring element using the concept of erosion and dilation. It checked in each pixel, how does this shape matches or misses local shapes in the image

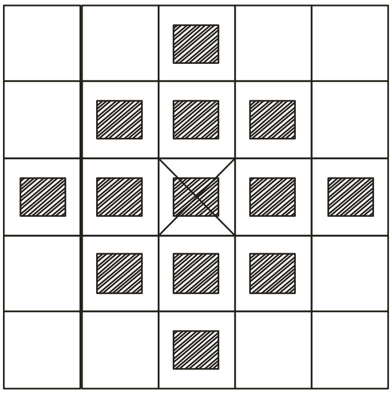
Structuring element— In the binary image the structuring element or probe is a small binary image i.e. a set or matrix of pixels, each with value of one (1) or zero (0).

- The size of structuring element specified by dimensions of matrix.
- The shape of structuring element specified by pattern of ones and zeros.
- One of the pixel of structuring element serves as origin.

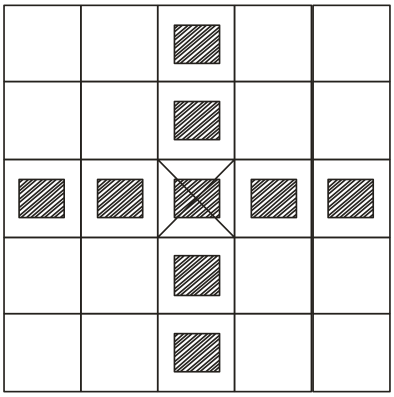
Few examples of structuring elements are—



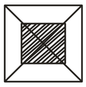
Square 3×3 element



Diamond shaped 5×5 element

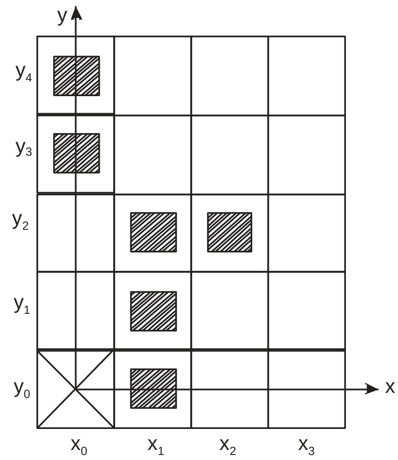


Cross-shaped 5×5 element



This pixel reffers to origine

Let an example of a point set in z^2, such as

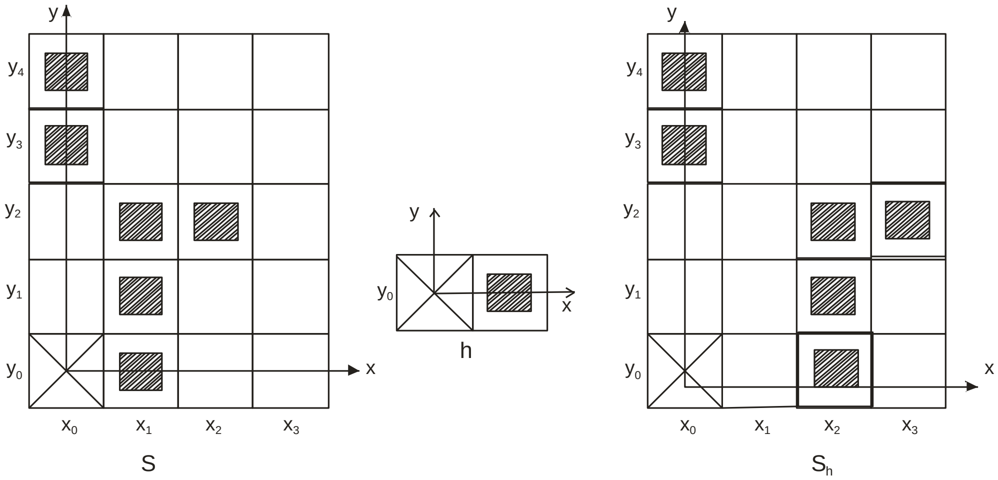


$S=\{(x_1,y_0), (x_1,y_1), (x_1,y_2), (x_2,y_2), (x_0,y_3)\ (x_0,y_4)\}$ (A)

Translation of a set by radiusuctor—

If translation S_h of a point set S by a radius vector h then

$S_h = \{p \in Z^2, P = x + h \text{ for some } x \in S\}$ 1(a)



$S_h = \{(x_2, y_0), (x_2, y_1), (x_2, y_2), (x_3, y_2), (x_1, y_3)\ (x_1, y_4)\}$ 1(b)

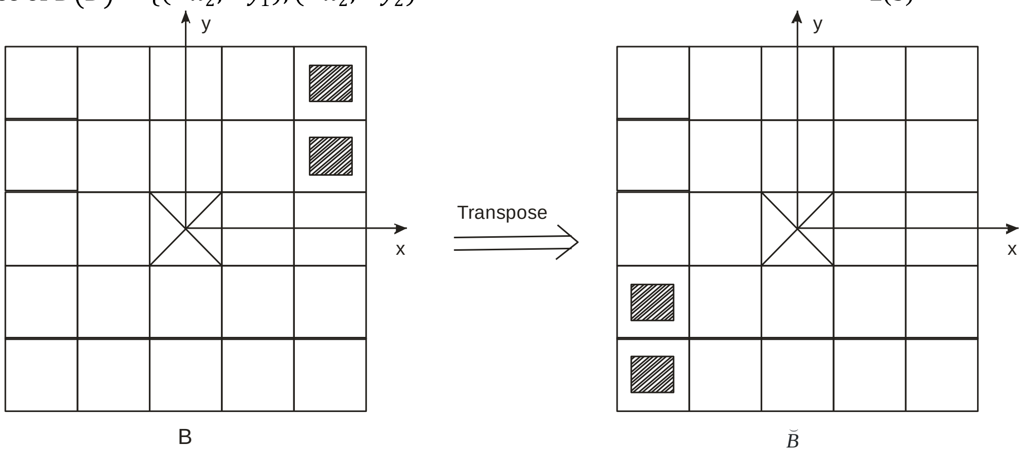
Transpose point set—

The transpose point set is defined as

$$\breve{B} = \{-b : b \in B\} \quad 2(a)$$

Let B $\{(x_2\ y_1), (x_2, y_2)\}$

Transpose of $B(\breve{B}) = \{(-x_2, -y_1), (-x_2, -y_2)$ 2(b)

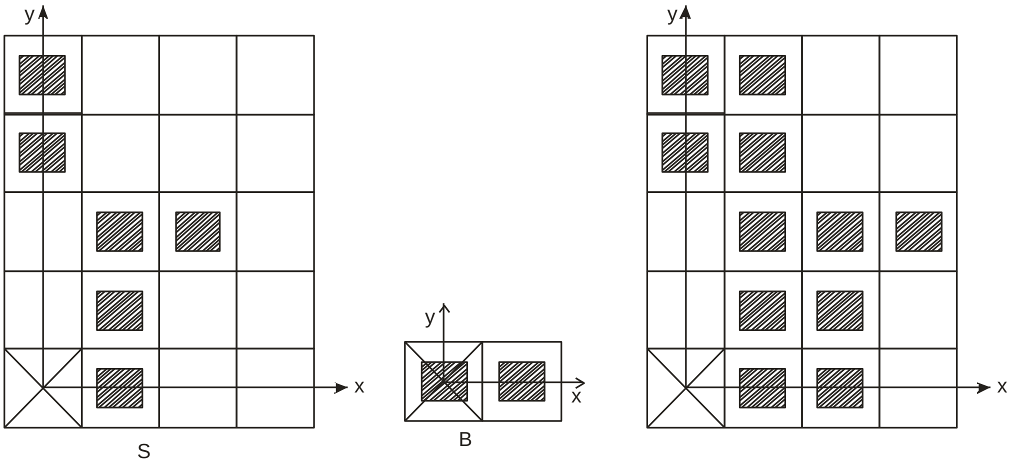


Binary Dilation— Dilation is the process of Minkowski addition and defined as—

$$\delta_B(S) = S \oplus B = \{P \in E^2 : P = x + b, x \in S \ \& \ b \in B\} \quad 3(a)$$

The dilation process increases object in the image.

Illustration :



$$S = \{(x_1, y_0), (x_1, y_1), (x_1, y_2), (x_2, y_2), (x_0, y_3), (x_0, y_4)\} \quad 3(b)$$

$$B = \{(x_0, y_0), (x_1, y_0)\} \quad 3(c)$$

$$S \oplus B = \{(x_1, y_0), (x_1, y_1), (x_1, y_2), (x_2, y_2), (x_0, y_3), (x_0, y_4)$$
$$(x_2, y_0), (x_2, y_1), (x_2, y_2), (x_3, y_2), (x_1, y_3), (x_1, y_4)\} \quad 3(d)$$

Dilation property—

(a) $S \oplus B = B \oplus S$ [commutative]

(b) $S \oplus (B \oplus C) = (S \oplus B) \oplus C$ [Associative]

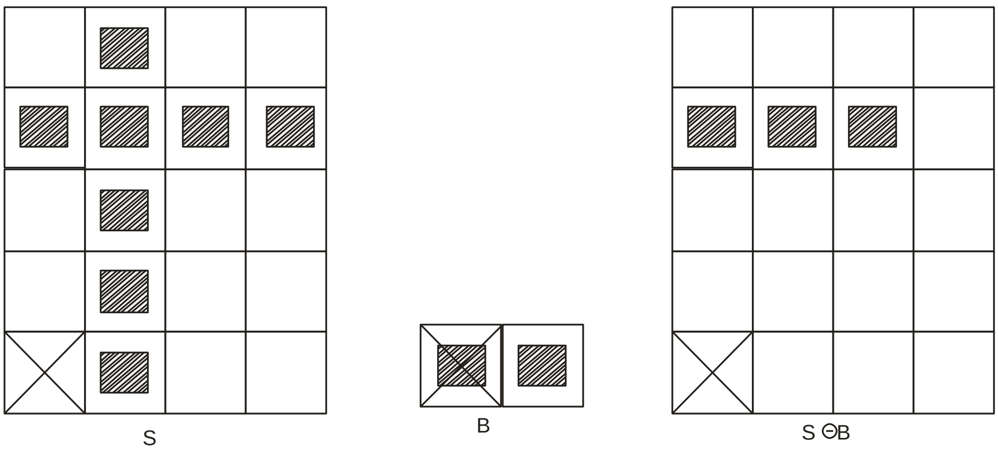
(c) $S_h \oplus B = (S + B)_h$ [Invariant to translation]

Binary erosion— Erosion is a process of Minkowski subtraction which is defined as—

$$\xi_B(S) = S \ \Theta \ B = \{P \in E^2 : P = x + b \in S \ \ for \ all \ \ b \in B\} \quad 4(a)$$

The erosion process reduces object in the image

Illustration :



$$S = \{(x_1, y_0), (x_1, y_1), (x_1, y_2), (x_0, y_3), (x_1, y_3), (x_3, y_3), (x_1, y_4)\} \quad 4(b)$$

$$B = \{(x_0, y_0), (x_1, y_0)\} \quad 4(c)$$

$$S \ \Theta \ B = \{(x_0, y_3), (x_1, y_3), (x_2, y_3)\} \quad 4(d)$$

Erosion properties :

(a) if $(0,0) \in B$ then $S \ \Theta \ B \subseteq S$

(b) $S_h \ \Theta \ B = (S \ \Theta \ B)_h$

$S \ominus B_h = (S \ominus B)_h$

(c) If $S \subseteq Q$ then $S \ominus B \subseteq Q \ominus B$

(d) $(S \ominus Q)^C = S^C \oplus \breve{Y}$

Used CT Image History and Structuring Elements: This sample MRI image has been procured from https://medpix.nlm.nih.gov/case?id=42380ba2-cd27-49af-8e19-7daa835f79a5&quiz=t having diagnosed with Meningioma Dermoid which is well defined extra axial mass on anterior flax on right. This sample image is processed as per mathematical model using different structural elements as shown below.

$$\begin{matrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{matrix}$$

Structure Element 1

$$\begin{matrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{matrix}$$

Structure Element 2

$$\begin{matrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{matrix}$$

Structure Element 3

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{matrix}$$

Structure Element 4

$$\begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}$$

Structure Element 5

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

Structure Element 6

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Structure Element 7

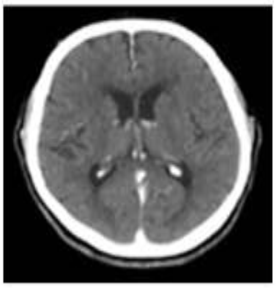
$$\begin{matrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{matrix}$$

Structure Element 8

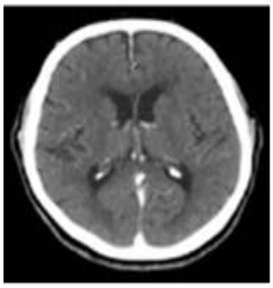
$$\begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{matrix}$$

Structure Element 9

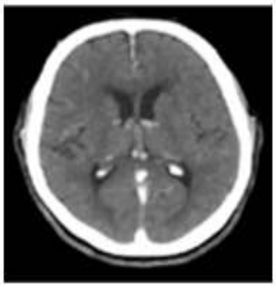
1-Images after Dilation Operation with pre-defined Structuring Elements-



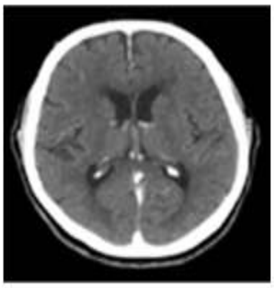
Dilation with Structure Element-1
Fig.-1



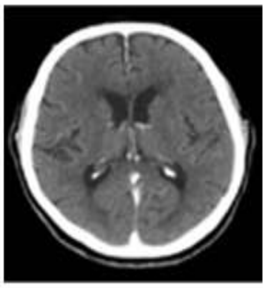
Dilation with Structure Element-2
Fig.-2



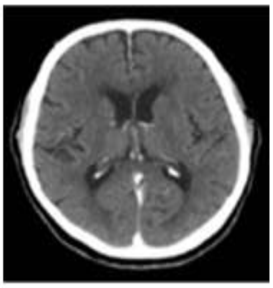
Dilation with Structure Element-3
Fig.-3



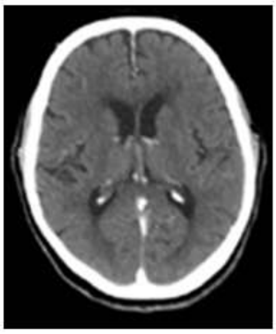
Dilation with Structure Element-4
Fig.-4



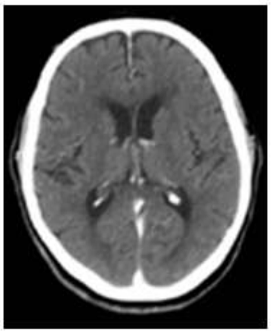
Dilation with Structure Element-5
Fig.-5



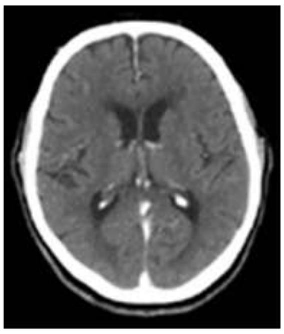
Dilation with Structure Element-6
Fig.-6



Dilation with Structure Element-7
Fig.-7



Dilation with Structure Element-8
Fig.-8



Dilation with Structure Element-9
Fig.-9

BW version of Erased Images-

Dilation with Structure Element-1
Fig.-10

Dilation with Structure Element-2
Fig.-11

Dilation with Structure Element-3
Fig.-12

Dilation with Structure Element-4
Fig.-13

Dilation with Structure Element-5
Fig.-14

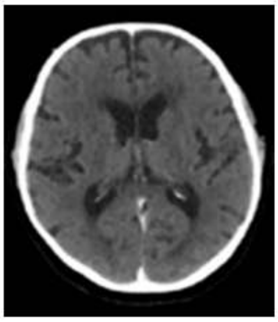
Dilation with Structure Element-6
Fig.-15

Dilation with Structure Element-7
Fig.-16

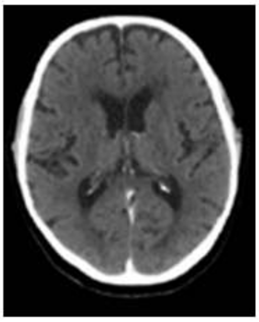
Dilation with Structure Element-8
Fig.-17

Dilation with Structure Element-9
Fig.-18

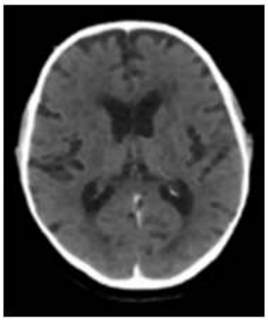
2-Images after Erosion Operation with pre-defined Structuring Elements-



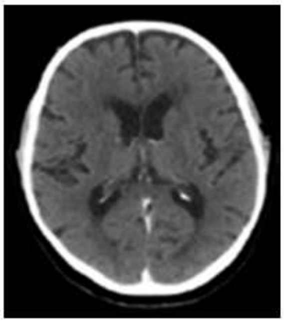
Erosion with Structure Element-1
Fig.-19



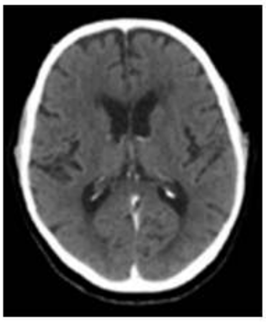
Erosion with Structure Element-2
Fig.-20



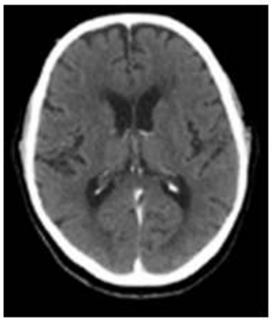
Erosion with Structure Element-3
Fig.-21



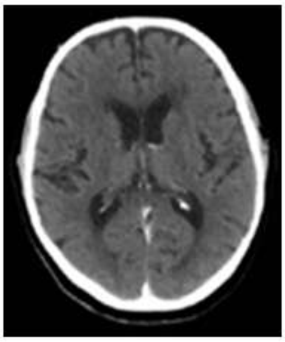
Erosion with Structure Element-4
Fig.-22



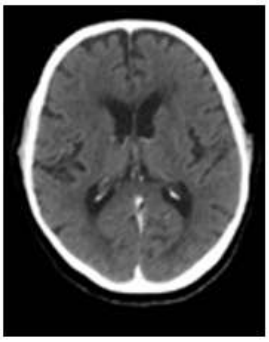
Erosion with Structure Element-5
Fig.-23



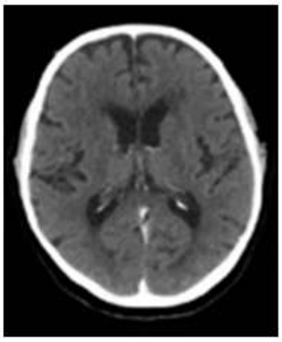
Erosion with Structure Element-6
Fig.-24



Erosion with Structure Element-7
Fig.-25



Erosion with Structure Element-8
Fig.-26



Erosion with Structure Element-9
Fig.-27

BW version of Erased Images-

BW version of Erased Image-1
Fig.-28

BW version of Erased Image-2
Fig.-29

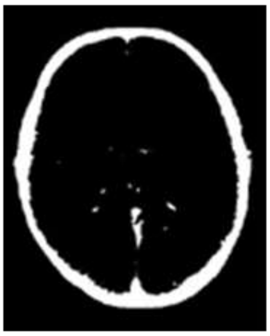
BW version of Erased Image-2
Fig.-30

BW version of Erased Image-4
Fig.-31

BW version of Erased Image-5
Fig.-32

BW version of Erased Image-6
Fig.-33

BW version of Erased Image-7
Fig.-34



BW version of Erased Image-8
Fig.-35

BW version of Erased Image-9
Fig.-36

Results and Conclusion:

In this paper we have discussed mathematical model of erosion and dilation operation with various structural elements to find out probable shape, size, and boundary of tumor, here we are testing effect of various structuring elements on the shape, size and edge of tumor for better diagnosis , the testing results shows that different shaped structuring elements significantly effects the shape , size and edge of tumor .Among tested structuring elements used here in dilation and erosion operation, structuring element-3 (diamond shaped 5x4 metrics with zero's forming diamond shape) shows most significant edge detection in dilation operation as obtained in figure-3 and its BW version which is in figure-6, where as structure element-6 (a 2x2 diagonal metrics) provide significant result in erosion operation represented in figure-24 and its BW version in figure-33. Thus we conclude that among tested probes, the structuring element-3 useful for extraction of probable shape, size and edge detection of tumor under dilation operation while structuring element-6 useful for same purpose under erosion operation.

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